Conversation: Sustainable Information Exchange Among Competitors

Bachelor Thesis

Department of Economics

University of Mannheim

submitted to:
Prof. Takakazu Honryo, Ph.D

submitted by:
Ichio Aoki

Student ID: 1499269
Degree Programme: Bachelor of Science in Economics (B.Sc.)

Address: Johann-Zimmermann-Weg 9, 53343, Wachtberg, Germany
Phone: (+)49 176 7277 5068
E-Mail: iaoki@mail.uni-mannheim.de

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List of Symbols

This is a list of variables used in section 3, with some explanations thereof

i, j	The game is played between two agents. i denotes the agent who's action is being observed while j denotes the opposing agent.
t	The period index. Conversation takes place from period $t = 1$, while period $t = 0$ is the investment stage for both agents.
θ	The degree of competition between the agents, with $0 < \theta < 1$. A higher value implies a higher degree of competition
β	A parameter appearing in both the utility function and cost function, with $0 < \beta < 1$. A high β means that conversation is gradual, as each step of the exchange adds little absolute value but has high relative gains. Also implies lower costs.
s_i	The idea that an agent holds. When observing the decision of an agent, $s = t$, as the idea version will correspond to the period.
$v(s)$	The utility derived from idea s.
p_i	The success probability, which can be interpreted as the skill of an agent.
$c(p)$	The cost function associated with the success probability. This represents an investment by the agent in order to attain a skill level p.
m	A parameter used in the cost function. Acts as a scaling factor, with a high value implying higher costs in general. Also serves as the maximum cost that can be incurred when investing.
E_i	The expectation an agent has over the opposing agent's success probability or skill.
$EU_i^t(\cdot)$	The expected utility of an agent in period t from sharing an idea with the opposing player, conditioned on both players always sharing after period t. Arguments are one's own success probability, one's expectation of the opposing success probability as well as the period t, θ and β.

1 Introduction

Conversation can be thought of as a cooperative or collaborative exchange of information between two or more parties. As social creatures, humans are inclined to communicate with others in their vicinity. In many such situations, conversations with friends or family members for instance, it is natural to expect both parties to communicate truthfully, as there is no incentive to do otherwise. An example for such an open and truthful conversation could be when asking family members for advice. Assuming no misaligned incentives, it is easy to believe that the advice-seeker shares all relevant information with the conversation partners and that these partners in turn reveal their true opinions and offer what they believe to be helpful insights.

However, introducing competition between the conversing parties may create incentives to not participate in the truthful exchange of information or to behave strategically. For example, two co-workers will most probably collaborate on a work project when there are no strategic tensions. Yet if a single senior position opens within their firm and they are faced with the possibility of a promotion, they may think twice before engaging in collaboration with their colleague, who has suddenly become a rival regarding the promotion. This example illustrates how the introduction of competition may slow or hinder the collaborative process.

Despite this intuition, there is plenty of evidence for conversation (henceforth assumed to be the exchange of ideas between agents) occurring truthfully in highly competitive environments within the economy. Empirical findings range from correlated trades between fund-managers to firms exchanging knowledge with each other and scientific exchange between academics. These observations beg the question, how such seemingly irrational exchanges can occur and be sustainable.

Theoretical literature on conversation is rather scarce; despite the abundance of research into one-sided communication with separate senders and receivers (e.g. Crawford & Sobel (1982)), bilateral communication, in which players are simultaneously senders and receivers, has yet to receive much attention. Stein (2008) introduced a basic model of bilateral conversation and several extensions have since been employed, but there is still much progress to be made in this field, especially when considering that most communication that takes place is bi-directional.

This thesis aims to provide an understanding as to how a truthful conversation can arise and remain sustainable between competitive collaborators. Scientific progress and more generally the advancement of knowledge is highly dependent on collaborative processes, making it important to understand the factors influencing the individual decision to embark on such processes. These two aspects are related to many positive externalities and therefore, analyzing any factors influencing collaboration decisions may be helpful in understanding how to further enable such exchanges.

The skill or ability of the conversation partners is thought to be one of the central aspects influencing conversational sustainability (Stein (2008), Ganglmair et al. (2016)). Two highly competent firms are more likely to develop a new groundbreaking technology than two incompetent firms with limited capacities. Previous models, such as Stein (2008), have incorporated this skill as an exogenous variable. Yet this premise of exogeneity is unrealistic, as in most cases, an agent's skill or ability is a result of costly investment (time, money, etc.). The contribution of this thesis lies in endogenising this factor of skill by extending Stein's basic model and making skill an endogenous, costly variable. The results are that a conversation can arise and be sustainable, despite this introduced strategic aspect of costly investment prior to the exchange occurring.

The thesis is organized as follows: Section 2 will present existing literature documenting collaboration in competitive fields and present factors vital for conversational sustainability. In section 3, I will briefly elaborate upon existing theoretical literature before introducing an extension of Stein (2008), incorporating an endogenous investment decision. The results of this extension will be summarized and discussed in Section 4. Section 5 will conclude this thesis.

2 Literature

The transmission of information between competing agents has been an observation for some time (von Hippel (1987), Schrader (1991), Appleyard (1996)). Any process of information exchange among competitors can be thought of as a collaborative process between two agents and be analyzed as such. It is reasonable to assume that many of these exchanges take place in the form of conversation. This transmission seems puzzling at first, as it is intuitive to believe that rational agents will not fully cooperate with their competitors. The following sections will present existing literature of examples for such collaboration in competitive environments and of the factors influencing the agent's decision to embark upon conversation.

2.1 Observing Conversation

In academia, work and research by teams is starting to dominate over that by individuals and the fundamental way research is conducted has seemingly become more collaborative (Wuchty et al. (2007)). Papers by single authors continue to constitute a lower percentage of overall research output compared to past decades (West et al. (2013)). This is true for all types of academic cooperation, be it within a certain scientific field (Hunter & Leahey (2008), Wuchty et al. (2007), Freeman et al. (2014)), cross-disciplinary, cross-institutional (Jones et al. (2008)) or cross-national (Leahey (2016)). Collaboration in the form of informal advice, data sharing or other forms of conversation (Lewis et al. (2012)) are not captured in these studies, implying that the amount of collaboration may be underestimated.

It is not only the number of collaborations that has increased over time but also the number of collaborators in a project (Wuchty et al. (2007)). All of this is despite the fact that competetiveness in the field of academia has seemingly increased over time (Larson et al. (2013), Cretsinger et al. (2003)).

Cooperation and information exchange is not limited to academia. The world of finance is known to be extremely competitive with considerable capital and effort spent in order to attain the 'best' investment ideas to attract customers. Managers tend to be rated relatively to one another, implying that the incentive to outperform peers is large. These characteristics have led to the fund-managers being labelled as cutthroats (Ganglmair et al. (2016)), willing to do anything in order to gain an edge over their competitors. Despite this somewhat negative characterization of the financial industry, professionals have been known to be open with their positions and trades. Many traders publicize their positions and the phrase 'talking your book' literally means that traders openly express thoughts and positions.

Studies have established that fund-managers and investors with various social ties exhibit correlated trading behavior. This behavior among managers working close to each other (Hong et al. (2005) and living close to each other (Pool et al. (2013)) presents strong evidence for the verbal transmission of financial ideas amongst fund-managers. Such transmission is also present among neighboring individual stock investors, with up to one half of this effect attributed to the verbal communication of ideas (Ivković & Weisbenner (2007)). Feng & Seasholes (2004) observe a similar pattern in investing behavior among individuals in China. Social ties may not be limited to being neighbors. Fund-managers who went to the same college together and are thereby more likely to be in contact with each other exhibit similar behavior (Cohen et al. (2008)). Hedge-fund managers with no geographic or social proximity are observed to directly share investment related information with each other privately on internet portals (Gray et al. (2012), Crawford et al. (2017)). These results all stem from connections or platforms which are likely to induce direct conversation between agents.

2.2 Conversation as a Trade-Off

Any conversation or collaboration entails inherent trade-offs. Sharing information with a rival leads to a direct loss of an information advantage and may also lead to misappropriation of the shared information. The opposing agent may choose not to reciprocate or share false information in turn. These risks must be weighed against the potential gains from sharing any information. Sharing could allow for the beginning of a mutually beneficial relationship (reciprocity in future Appleyard (1996), Schrader (1991)) or lead to an efficiency gain through a collaboration. Factors influencing both the costs and benefits associated with this trade-off have been the focus of research in this field.

Competition

One of the central aspects in the individual decision to embark on a conversation is the degree of competition between the agents. Bikard et al. (2015) find that cross-departmental collaboration among scientists leads to the highest productivity gains and most fruitful results. This may be explained by the hypothesis that researchers in different departments are subject to a lower degree of competition than two researchers in the same or overlapping fields and are therefore willing to engage in more open and fruitful conversation. Academic scientists are less willing to share information regarding a problem or question with others if the prize associated with solving the problem is large (Haeussler et al. (2014)). This can be seen as a proxy for the degree of competition between the agents. A higher degree of competition implies a higher potential loss from sharing with the opposing agent.

Skill

The abilities of the collaborators also play a significant role in the decision to share information as well as the length of a conversation. This is intuitive as the potential gain from conversation is higher the more skilled the opposing agent is. The expected length of a conversation and the quality of the resulting final product is also positively dependant on the skill of the participating agents (Ganglmair et al. (2016)).

Scientists are more likely to share information with each other if the opposing scientists' results are more valuable and if the possibility of using these opposing results is high (Haeussler et al. (2014)). Ganglmair et al. (2016) find that conversation is more likely to be sustainable the higher the opposing players' skill level related to the topic of conversation is. Reputation, a proxy for skill, is also found to positively correlate with collaboration, such as in the cases of academic prestige of the institutions an agent is associated with (Hunter & Leahey (2008)) or investor reputation (Liu (2017)).

It is not only absolute levels of skill that play a role. Ceteris paribus, those with similar ability are more likely to share than those with a large difference in skill (Haeussler et al. (2014)). In the world of finance, fund managers may be sharing information with each other as they are aware of each other's skill as a professional in their field. Araújo et al. (2014) find that researchers who are more alike in terms of productivity and being scholarship recipients tend to collaborate more with each other.

In academia, one of the more cited drivers of collaboration is specialization, with more scientists becoming specialized and it being ever harder to single-handedly tackle questions (Hara et al. (2003), Jones et al. (2008)). It is well-known that specialization and division of labor can lead to efficiency gains (Leahey (2006)). This could be a driver influencing both the skill of individuals as well as the competitiveness. Specialization may be increasing individual skill levels and decreasing competition, positively influencing the decision of agents to collaborate with others.

Specialization may also increase the overall need for collaborative processes. As one increasingly specializes in one area, the dependence on other specialists in other fields to tackle a whole area of study may increase. This may imply that the degree of complimentarity between scientists has increased over time which has been found to play a vital role in conversations (Stein (2008), Hellmann & Perotti (2011)).

Quid-pro quo

Another compelling reason to engage in conversation with a competing agent is the expectation of reciprocity or quid-pro quo (Pool et al. (2013), Ganglmair et al. (2016)). Granting access to one's materials or information today may induce the opposing party to do so in the future, while denying such a request may jeopardize a relationship for an unforeseeable amount of time. If an agent expects the conversation partner to be likely to share information in the future, he may reveal something now in order to maintain a healthy relationship. One's probability of sharing with others may then be increasing in the expectation of others doing so. Pool et al. (2013) assert that this, along with the fact that the direct information costs of sharing are relatively small, is a likely factor influencing fund-managers to share financial information with their neighboring managers.

Feedback

The possibility of receiving feedback from another party may also be motivating agents to converse with their competitors. In order for feedback on an idea to be valuable, it must come from someone with proficiency in the same field. Yet this may mean that the only way to receive feedback is from sharing with competing agents. Haeussler et al. (2014) survey scientists regarding their potential decision to share materials and other research relevant information with their peers. They find that both the expectation of future reciprocity (quid pro quo) as well as the potential for feedback primarily drive researchers to share with others. Gray et al. (2012) and Crawford et al. (2017) find that the possibility to receive feedback is a vital aspect of communicating financial 1ideas.

2.3 Theories of Conversation

On the theoretical side of sustained, multi-period conversation or bilateral communication, literature is limited. Stein (2008) introduces a model of conversation consistent with the empirical observations above. Conversation is found to be more likely the lower competition between agents is and the higher agent's skill levels are. Strict complimentarity is a necessary condition for conversation to materialise. Stein's model also brings with it implications for ideas resulting from conversations; the value of an idea can be measured by the distance it has travelled, with good ideas being kept in the hands of few. This is consistent with observations that larger research collaborations appear to be less advantageous (Bikard et al. (2015)), as well as other theories on the value of information (Manela (2014)). Phenomena such as economic bubbles or scams (Rantala (2017)) may be explained by such a theory of communication. Crawford et al. (2017) assert that investors wish to keep discussion of the very best ideas among themselves and limit their spread.

Some theoretical extensions of Stein (2008) have since been proposed. Ganglmair et al. (2016) confirm the importance of both parties' skill on the decision to participate in conversation. Expectations of reciprocity also positively incite agents to share. Ganglmair & Tarantino (2014) investigate the role of payoff relevant private information in the conversational framework. Information revelation may occur without the need of contracts or mechanisms out of fear of early conversation termination due to the rent-extracting capabilities of the agent endowed with the private information. The general sustainability of a conversation is negatively impacted by the mere possibility of a secret existing between the two conversing parties.

Further models have been developed with assumptions differing from the complimentary design found in Stein (2008). In Rosenberg et al. (2013), agents engaging in repeated games with private information can, if faced with unrelated decision problems, engage in mutually beneficial information exchange. Horner & Skrzypacz (2009) and Blume & Park (2015) further show that information exchange can occur in strategic settings between competing rational agents.

To summarize this section, conversation is observed to occur between agents in many competitive environments. Evidence points to several factors which influence the individual decision to embark upon such a process, including the skill of the parties involved or expectations of future reciprocity. Several theories supporting these hypotheses exist, although there is still potential for further research in this field. This includes developing more models to support the observed phenomena and empirically testing the hypothesis of conversation between competitors.

3 Model of Conversation

Previous models such as Stein (2008) take into account the costs of sharing as the loss of informational advantage. Ganglmair et al. (2016) allow for an asymmetrical modification of this factor and find that even with differing skill levels, there are situations in which agents will collaborate with each other. The results state that low competition, high skill and high relative incremental benefits from conversation are favorable to conversational sustainability. Skill level is also a key factor influencing conversation length which in turn affects the total gain from any conversation.

However, in most cases, skill or ability is not an exogenous parameter, as presumed in these models. A firm will have a high ability because it invested in production capacities or human resources at a substantial cost. A scientist's expertise in a given field comes from the time spent researching and studying. While skill is to a certain degree an intrinsic quality of the agents, a substantial part of it is also the result of investments in the form of money, time or other opportunity costs.

The extension proposed in this thesis attempts to endogenise the skill each agent has by introducing a stage of investment prior to the conversation occurring. This simplifies the assumption of costly skill acquisition somewhat; assuming that the entirety of an agent's skill regarding the collaborative process he is facing is derived from an investment made prior to the conversation occurring. One way of interpreting this approach is that an agent commits to an effort level in a conversation prior to the process beginning.

I am interested in establishing whether there exists an equilibrium strategy of setting a skill level which remains compatible once the conversation has started. It is reasonable to believe for example, that an agent has an incentive to free-ride if he knows the opposing agent will invest in skill (e.g. be part of a group project without contributing). Perhaps an agent would like to keep the final pieces of the joint project to himself in order to extract some rent from the opposing agent (e.g. keep the final proof of a formula to oneself in order to publish it as a sole author later), or not participate at all due to uncertainty. Given the existence of these possible actions by agents, the sustainability of exchanges must be analyzed in this context.

The situation analyzed in the model can be summarized with an example of a possible collaboration between firms A and B. A will take into account the degree of competition as well as the benefits of collaborating with B and set a plan that maximizes its profits from the project given every single skill level B could have. This plan consists of setting an optimal level of investment which translates to a skill level. A also knows that B is doing the same. As both A and B know the opposing player's strategy, they deduce their optimal point of action and evaluate whether selecting this action forces commitment to collaboration (Incentive compatibility constraint (IC)). They finally check whether this course of action is profitable and better than not participating (Individual rationality constraint (IR)). If the equilibrium decision fulfills both the IC and IR conditions, they will both take the deduced action and engage upon the collaborative process.

I limit the focus to symmetrical equilibrium points by making the participating agents ex-ante identical. By introducing heterogeneous agent characteristics, this extension can be extended to include asymmetrical equilibria. The results remain consistent with previous models by Stein (2008) and Ganglmair et al. (2016); competition negatively affects conversational sustainability while higher incremental gains from conversation encourage it. These parameters also affect the equilibrium skill levels which in turn positively affect conversational sustainability. Higher costs associated with the investment negatively affect an individuals' investment decision, threatening conversational sustainability.

The basic model, specifically the setup, borrows from Stein (2008) as well as the asymmetrical extension from Ganglmair et al. (2016) and will be kept relatively short. After introducing the basic model setup and conditions needed in equilibrium for conversation to arise, the investment stage and individual investment decision will be introduced. Characteristics of the resulting equilibria and limitations of the model will then be discussed. The derivations of some of the formulas can be found in the appendix.

3.1 Model Setup

Market Structure

The game takes place between two agents, agent 1 and agent 2. Each agent faces a unit market, with an overlap between the two markets denoted by θ, with $0 < \theta < 1$. This parameter represents the degree of competition between the two agents. Each agent holds a monopoly on their part of the market, $1 - \theta$, and they compete in price following the Bertrand principle over the overlapping segment, as in figure 3.1.

Ideas and Payoffs

An idea is denoted by the variable s, where the first idea is $s = 1$ and the nth idea is $s = n$. Each idea is to be thought of as a successive improvement upon the previous idea and an idea $s = n$ includes all the information of the previous $n - 1$ ideas.

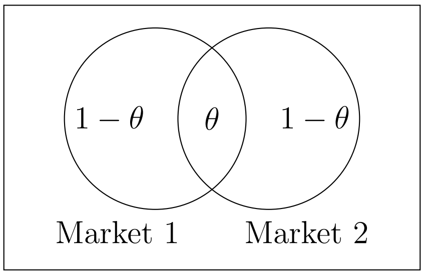

Figure 3.1: Market Structure

A tangible example of this concept could be a technology in development, where n represents the version of the technology in question.

An agent can take the latest idea endowed to him and convert this into a payoff with the function $v(s)$. Adhering to the example above, this process can be thought of as using the new technology to lower production costs or bring a new improved project to market. $v(s)$ depicts this as a gain, that is the net benefit one has from inputting $s = n$ into the payoff function. It must therefore be of such form that $v(0) = 0$, as in this scenario nothing has been gained compared to before the conversation. It must also have increasing returns, that is $\frac{dv}{ds} > 0$. In the following analysis, assume the functional form

$$v(s) = 1 - \beta^s, \ 0 < \beta < 1$$

which has the added property of diminishing returns.[1]

The intuition behind β is as follows: β represents the gain from an incremental idea, that is a high β implies a higher relative gain from conversation and a slow decline in benefit from incremental ideas. A high β therefore adds a gradual nature to the conversation and can also be interpreted as a degree of complexity of the conversation, with more complex processes (higher β) requiring much longer conversations in order to develop into large payoffs. β will henceforth be thought of as the speed of the conversation, with higher values implying slower progression.

This can be better understood using an example. It is clear that for all $\beta \in (0, 1)$, $v(s) \to 1$ if $s \to \infty$. A variation in β changes the speed with which this happens. If one takes $\beta = 0.99$, the first idea brings a benefit of $v(1) = 0.01$, the second a benefit of $v(2) = 0.0199$, an increase of 99%. Taking $\beta = 0.1$ leads to $v(1) = 0.9$, $v(2) = 0.99$, an increase of only 10%. Yet in the second case, the lower value for β leads to faster absolute value from the conversation, meaning less iterations are needed for any given benefit. And so, high β can be interpreted to mean a gradual

[1] Specifically, the gain from one idea at period t is $v(t+1) - v(t) = \beta^t(1-\beta)$, which is decreasing in t.

process with low contributions but high relative gains.

Conversation

In period $t = 1$, one of the players is randomly selected and endowed with the initial idea $s = 1$. Assume, without loss of generality, that this is player 1. 1 has a choice between keeping the idea to himself or sharing it with the other agent, player 2. Keeping an idea to oneself leads to the termination of the conversation and payoffs are realized. If the idea is shared, the next period $t = 2$ begins, where player 2 has a probability p_2 of coming up with a new idea $s = 2$. If 2 is unsuccessful, the conversation ends and payoffs are realized. If he is successful, he is faced with the same choice as player 1 in the previous period to share idea $s = 2$. This process continues indefinitely until either one player conceals an idea or fails to come up with the next idea.

Once conversation has ended, either by concealment or failure to come up with the next idea, each player takes the latest idea endowed to him which is denoted as s_i and converts it into a payoff. Due to the nature of the markets, agents receive the total payoff

$$U_i = (1-\theta)v(s_i) + \theta \max\{0, v(s_i) - v(s_j)\}$$

with s_j denoting the number of ideas held by the other player. The first part of the term represents the monopoly segment of the respective market and the second term incorporates the Bertrand competition present in the overlapping market segment.

Investment and Probabilities of Success

Once an agent has been given an idea by the conversation partner, they must improve upon it in order for the next action to commence. The probability of success p_i can be interpreted as the competence or skill of an agent in regard to generating the next idea in the conversation. It is dependent on c_i, the investment decision of the player, undertaken in $t = 0$ before the initial exchange and is not known to the opposing player. This investment decision is denoted with the function $c = c(p)$ which must be convex over $p \in (0, 1]$. Setting $p = 0$ represents the outside option of non-participation and is associated with a direct payoff of 0 for both players.

3.2 Conditions for Sustainable Conversation

In order for conversation to materialize in any period, the expected utility of a player sharing an idea with the opposing player must be larger than the utility of concealing an idea. In any period $t \geq 1$, an agent is in possession of idea $s = t$. [2] Then, the payoff from concealing this idea and stopping the game can be expressed as

$$\begin{aligned} U_i^t(\text{conceal}) &= (1-\theta)v(t) + \theta(v(t) - v(t-1)) \\ &= (1-\beta^t) - \theta(1-\beta^{t-1}) \end{aligned} \tag{1}$$

as the other player is in possession of idea $s = t - 1$.

On the other hand, the expected utility of sharing in any period t is based under the assumption that both players always share in any period after period t. Since the expected utility is dependent on both agents' skill levels, an agent calculates it using an expectation of the opposing player's skill level. This is denoted by E_i. The expected utility of sharing in period t takes the form

$$\begin{aligned} EU_i^t &= (1-\theta)\sum_{k=0}^{\infty}(p_iE_i)^k[(1-E_i)v(t+2k) + E_i(1-p_i)v(t+1+2k)] \\ &= (1-\theta)(1-\frac{\beta^t(1-E_i+E_i(1-p_i)\beta)}{1-p_iE_i\beta^2}) \end{aligned} \tag{2}$$

The profit on the overlapping market segment θ is 0, since both agents always share and are thereby ensured to have the same amount of signals.

Using equations (1) and (2), the condition for conversation to arise in any given period t can be derived. This happens if the expected utility from sharing (2), is larger than the utility of concealing (1) and takes the form

$$EU_i^t \geq U_i^t(\text{conceal}) \Leftrightarrow \frac{1+\beta p_i}{1+\beta E_i}\beta E_i \geq \theta \tag{3}$$

This condition is independent of t and therefore consistent through time. As long as it holds for both players, conversation will be sustainable.

Conversational sustainability is more restricted the higher the degree of competition, θ, is. This is intuitive, as a higher degree of competition means that the benefit from conversation is restricted to a smaller monopolistic market segment and the one idea advantage over the opposing player has a larger benefit. The condition is less restrictive the higher p_i, E_i and β are. [3] The reasoning behind the first two

[2]This is after $s = t-1$ has been shared with the player, who has then successfully come up with idea $s = t$. Conversation is terminated if an agent fails to come up with an idea, so every decision made by the players is conditioned on having successfully come up with the next idea, which in turn is wholly dependent on the previous idea having been shared.

[3]For the derivatives of expression (3), refer to Ganglmair et al. (2016), by substituting E_i (expected skill) for $\sigma_i p_j$(expectation of sharing multiplied by skill).

effects is simple. The gains from concealing in any period are independent of the probability that further ideas may be conceived, [4] as the game will end in the period of concealment. On the other hand, the expected return of conversation is increasing in both probabilities, as the probability that the conversation will end due to failure to come up with a new idea is decreasing in these two variables.

β represents the relative gains from each step of conversation as well as the speed of the exchange. Condition (3) states that high βs lead to more sustainable conversation, which is intuitive as the relative gain from conversing is high while the cost (loss of information advantage) is low. As a high β implies a gradual nature of the conversation, this result implies that conversations with slow progression are more sustainable than faster ones.

3.3 Equilibrium

Symmetrical Equilibrium

In equilibrium, a player's belief about the opposing player's skill level must be consistent with the actual level of skill selected. This means that $E_i = p_j$ must hold. This extension will focus on symmetrical equilibria where both players select the same investment in period 0.

In the game, only one player can be endowed with the initial idea. The other player's first action, conditioned on having received the first idea s = 1 and successfully improved upon it, is to share or conceal the **second** idea. This means that the expected utility of the first action differs between both agents.[5] This issue is solved by randomizing the initial endowment which makes the agents solve their optimization problem over the expectation that they will be either first or second receiver with probability $\frac{1}{2}$ respectively. This makes both agents ex-ante identical and allows for a symmetrical equilibrium to exist.

Symmetry simplifies equation (3) to

$$p_i = p_j = p \geq \frac{\theta}{\beta} \tag{4}$$

as in Stein (2008). The interest lies in determining whether there exists an equilibrium investment decision in $t = 0$ for both players which fulfills constraint (4). This

[4] Of course, the period that has been reached and therefore the idea an agent is in possession of is dependent on these two variables, however as the agents decision is being made conditioned on the period t having been reached, they play no role in forming the utility from concealing in this period.

[5] The difference in utility between these two players can be seen in the derivation of the total expected utility function in period 0, which can be found in the appendix.

condition gives an upper-bound for θ, as $\theta \leq \beta$ must hold for any sort of sustainable conversation to arise.

Agents' Optimization Problem

Prior to the conversation, each player has the opportunity to invest an amount c and improve upon their own success probability. They choose this amount by maximizing their expected utility in t = 0, while assuming a given opposing probability, so that they have a best-response to each possible realization of the opposing agent's skill.

The agents have identical, symmetrical expected utilities in period 0, which can be expressed as follows:

$$EU_i^{t=0}(p_i; p_j) = \frac{1-\theta}{2}(2 - \beta\frac{2 + p_j(\beta - 1) + p_i(\beta - 2p_j\beta - 1)}{1 - p_i p_j \beta^2}) - c(p_i) \qquad (5)$$

Maximizing this function over p_i for every given opposing probability p_j supplies agent i's best-response curve (BRC). In order to continue the analysis, the cost function is specified to be

$$c(p) = m(1 - \beta)\frac{p^2}{1 - p\beta} \qquad (6)$$

for any positive m. This function was chosen in large part due to the simplification it brings to the later analysis. While it also brings with it some restrictions, it fulfills the criteria placed on the cost function at the model setup, as it is convex over $p \in (0, 1]$ and $c(0) = 0$. Investment is associated with an increasing cost up to a maximum of m for $p = 1$. This characteristic of a maximum cost may present an issue and will be discussed in the discussion section.

$\frac{dc}{d\beta}$ is strictly negative. That is, a higher β lowers costs associated with the investment, which is consistent with the gradual nature a higher value for this parameter brings to the conversation process. This means that a slower conversation is associated with low contributions in each period, which in turn require lower costs to acquire. The use of β in this function leads to a loss of generality, as there is an interpretation in which slow conversations may in fact be associated with higher general costs or that costs be independent of the speed of a conversation. These issues will be addressed in the discussion section.

Payoffs

After choosing the symmetrical optimal investment, the expected utility of both agents is calculated by inserting $p_i = p_j = p$ (symmetry) into equation (5) as follows:

$$EU(p) = \begin{cases} 1-\theta-m & \text{for } p=1 \\ (1-\theta)(1-\beta\frac{(1-p)}{1-p\beta}) - m\frac{p^2(1-\beta)}{1-p\beta} & \text{for } 1>p>0 \\ 0 \text{ as the outside option} & \text{for } p=0 \end{cases} \quad (7)$$

Individual rationality is ensured if $EU(p) \geq EU(0)$ for $p \in (0,1]$. This inequality can be simplified to the condition

$$(1-\theta)\frac{1}{p^2} \geq m$$

and must be checked for the equilibrium skill levels p chosen by the individuals. If this condition is not violated in equilibrium, the agents will participate in conversation. Non-violation of condition (4) then ensures the sustainability of the exchange.

3.4 Results

Best Response Curves (BRC)

The agent maximizes his expected utility given in expression (5) with regard to p_i and derives his best response curve $p_i(p_j)$. However, this equation is limited in both range and domain to the intervals $(0,1)$, as p_i and p_j represent probabilities and $p=0$ represents the outside option of non-participation, while $p=1$ means 'full' participation. Figure 3.2 illustrates this limitation. As the best-response curve is an implicit function of both p_i and p_j, the analysis is difficult. However, the following can be stated:

1. $\lim_{p_j \to 0} p_i(p_j) > 0$
2. The function is weakly increasing over $p_j \in (0,1)$ [6]

These two characteristics have several implications. The first is that no free-riding occurs in this situation. It is optimal to increase one's effort if the other does so, as the skill levels are strategic compliments. This arises in part due to the complimentary idea generation process. The second is that the result will be either a 'full' participation equilibrium at $p=1$ or an interior equilibrium at $p \in (0,1)$.

[6]This is derived from plotting the function and will not be analytically proven in this thesis

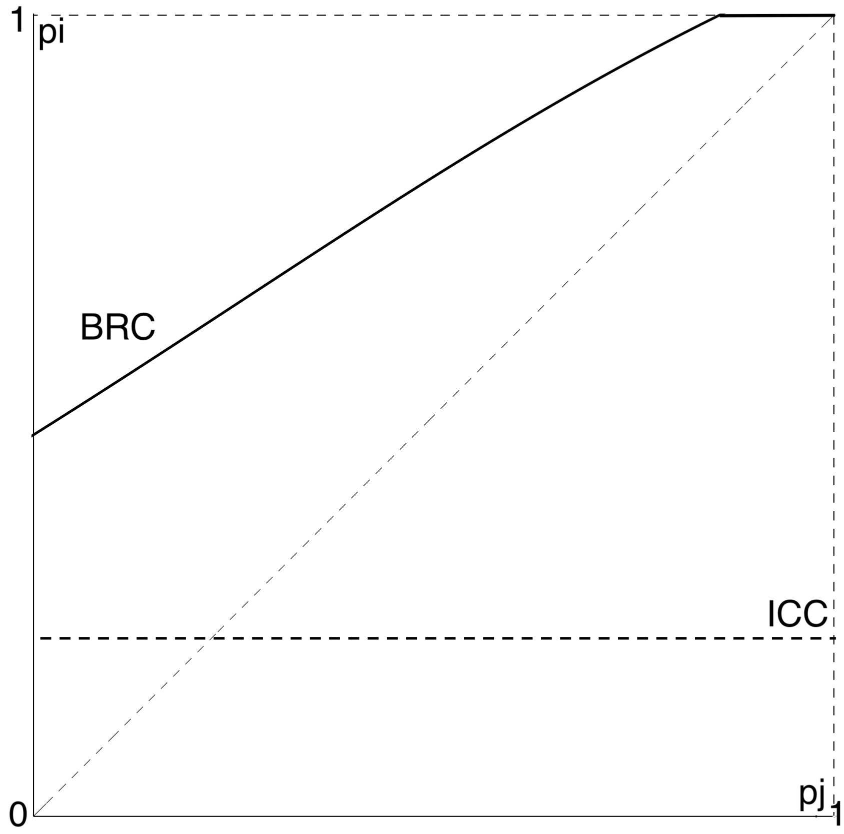

Figure 3.2: 'Full' participation with $\theta = 0.2$, $\beta = 0.9$ and $m = 0.1$

Equilibrium Points

The equilibrium can be calculated as the point where the agents' BRCs cross, which will occur at $p_i = p_j = p$ due to the symmetrical nature of the game. This can be represented as the BRC of an agent intersecting with the 45 degree line from the origin. Solving for these points yields the following results:

$$p_1 = 1 \tag{8}$$

$$p^* = \frac{2m - \sqrt{2m(\beta^2(\theta - 1) + 2m)}}{2\beta m} \tag{9}$$

with p_1 denoting a corner solution which will occur if the BRCs are cut off. The conditions for p_1 or p^*, the interior solution, occurring in equilibrium will be analyzed below.

Full participation

'Full' participation is meant when each agent chooses $p = 1$ in equilibrium. As long as condition (4) holds, agents will indefinitely successfully share ideas. An equilibrium of this sort is depicted in figure 3.2, where the dotted horizontal line represents equation (4), the incentive compatibility constraint (ICC). The condition for this occurring is :

$$m \leq \frac{(1-\theta)\beta^2}{2(1-(1-\beta)^2)} := \underline{m} \tag{10}$$

Checking the individual rationality (IR) constraint $EU(1) \geq EU(0)$ yields:

$$1 - \theta \geq m$$

which always holds in the case of condition (10), as $(1-\theta) > \underline{m} \geq m$.

Interior Solutions

In the case that equation (10) is violated, the game has an interior solution denoted $0 < p^* < 1$. The interior solutions is denoted by equation (9) and exists, as $m > \underline{m}$ implies that the sum under the root is positive. In order for the interior solution p^* to be an incentive compatible equilibrium, condition (4) must also hold. This yields the following inequality:

$$m \leq \frac{(1-\theta)\beta^2}{2(1-(1-\theta)^2)} := \overline{m} \tag{11}$$

These conditions can be combined to

$$\underline{m} < m \leq \overline{m} \tag{12}$$

which ensures an incentive compatible interior equilibrium, the point of primary interest. The individual rationality constraint must also be checked, that is $EU(p^*) \geq EU(0)$. Checking this condition yields the inequality

$$m \geq \frac{(1-\theta)\beta^2}{2}$$

which is fulfilled with condition (12). Every single interior solution is associated with a positive payoff and therefore, every interior solution fulfilling condition (12) is an incentive compatible, rational equilibrium.

Figure 3.3 depicts an incentive compatible interior equilibrium, as the symmetrical optimal decision point lies above the horizontal line representing equation (4), the IC constraint. Figure 3.4 shows an interior crossing of best-response curves, yet because the second part of inequality (12) does not hold, it is not incentive compat-

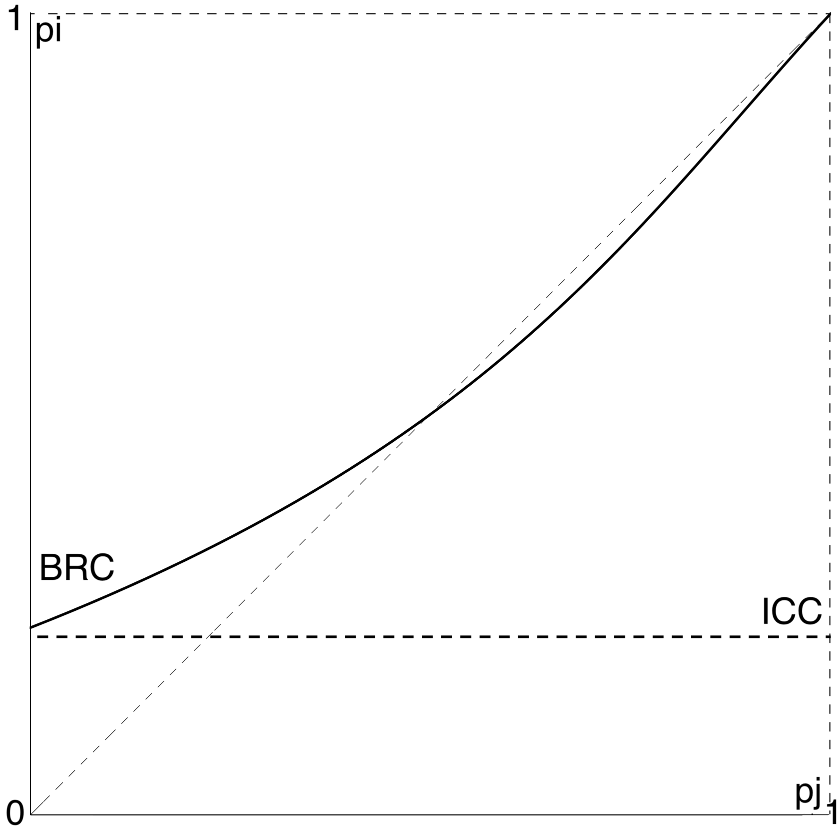

Figure 3.3: IC compatible interior with $\theta = 0.2$, $\beta = 0.9$ and $m = 0.3$

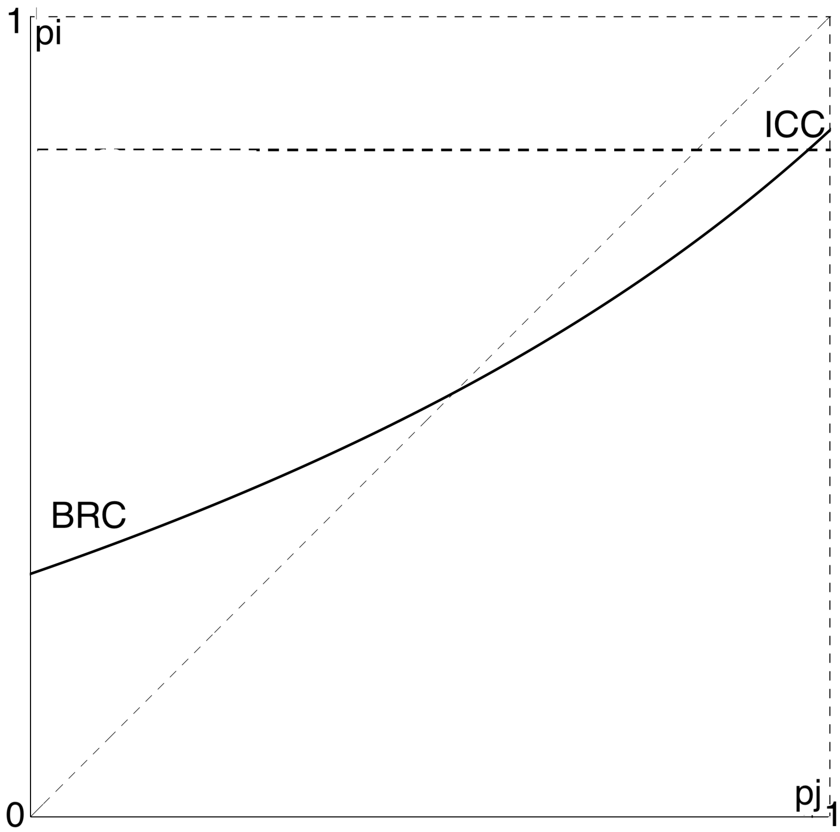

Figure 3.4: Non-viable interior with $\theta = 0.5$, $\beta = 0.6$ and $m = 0.1$

ible and can therefore not be an equilibrium of the agent's optimization problem.

Comparative Statics

I will now analyze the effects the parameters θ, β and m have on the equilibrium result. These parameters influence the type of equilibrium that occurs and in the case of an interior solution, the value of this solution as well as whether it complies with equation (4).

First, the partial derivatives of (12) with regard to θ yield the overall effect of an increase in θ.

$$\frac{\partial \underline{m}}{\partial \theta} < 0$$
$$\frac{\partial \overline{m}}{\partial \theta} < 0$$

The upper-bound value for 'full' participation which is also the lower-bound for an interior solution and the upper-bound for an interior, viable solution are decreasing in this parameter. This means, somewhat intuitively, that ceteris-paribus, an increase in competition will endanger conversational equilibrium, as 'full' participation is less likely with an increasing θ. Furthermore, in the case of an interior solution, the likelihood that the incentive compatibility constraint is fulfilled is decreasing in θ, which may result in no equilibrium solution.

The same analysis can be done for β, which leads to:

$$\frac{\partial \underline{m}}{\partial \beta} > 0$$
$$\frac{\partial \overline{m}}{\partial \beta} > 0$$

Increasing the parameter β which depicts relative incremental gains of conversation will lead to an increased likelihood of sustainability and may even lead to a 'full' participation equilibrium.

These properties are shown in figures 3.5 and 3.6 which depict the relationship between θ, β and the equilibrium decision for $m = 0.2$ and $m = 0.4$, with β on the x-axis and θ on the y-axis. The white areas represent non-viable points, as those above the 45 degree line represent $\beta \leq \theta$ and those beneath it are not incentive compatible (violation of (11)). The checkered area represents all incentive compatible, interior solutions, i.e. all points for which equation (12) holds. Finally, the dark area represents points of 'full' participation, as equation (10) holds for these points. The figures show that an increasing m results in the set of non-viable points increasing,

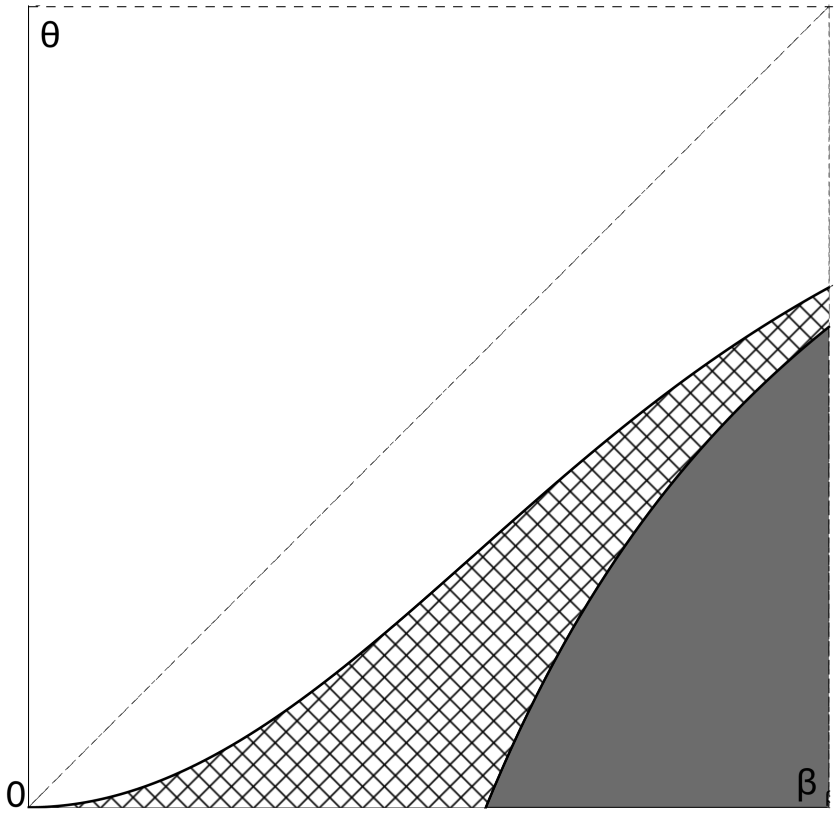

Figure 3.5: Plot of viable points for m = 0.2

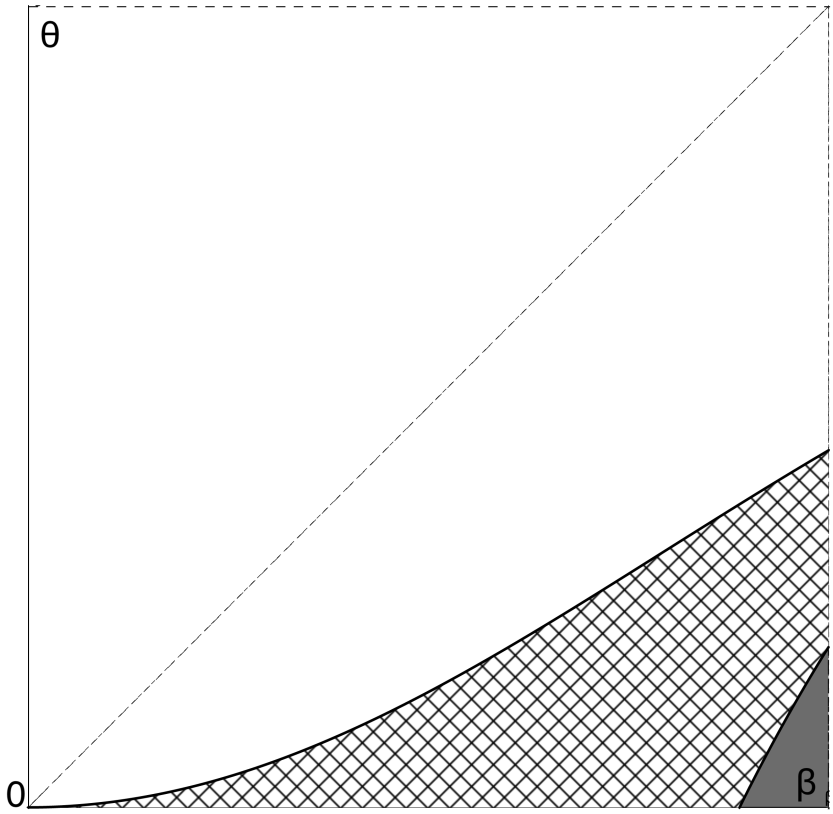

Figure 3.6: Plot of viable points for m = 0.4

the set of 'full' participation equilibrium pints shrinking and the set of viable interior points shifting down and to the right (lower θ and higher β needed).

The interior solution has the following characteristics:

$$\frac{\partial p^*}{\partial m} < 0$$
$$\frac{\partial p^*}{\partial \theta} < 0$$
$$\frac{\partial p^*}{\partial \beta} > 0$$

That is, in equilibrium, an agent will invest more the lower the costs of doing so, the lower the competition between himself and the opposing agent and the higher the relative benefits of conversation.

4 Discussion

4.1 Result Summary

The discussion will be limited to the interior solution. This is due to the questionable nature of 'full' participation, which will be elaborated on below. The results show that a sustainable conversation equilibrium is possible despite costly attainment of skill or ability. Decreasing competitiveness and costs or increasing the benefits associated with collaboration will lead to an increase in the investment. This is consitent with previous work by Stein (2008) and Ganglmair et al. (2016) in that conversational sustainability is negatively affected by competition and positively affected by the parameter β. Also, there is no free-riding in this setting, despite the intuitive incentive to do so (invest less since opposing agent invests). This is due to skill being a strategic compliment as well as the complimentary design of conversation in this model (increasing and non-zero BRCs).

4.2 Application

The expected length of a conversation is increasing in the agent's probabilities of success.[7] As a longer conversation implies a 'better' result[8] (Stein (2008)), it should be a priority to maximize the endogenous investment each agent inputs and thereby increase the expected length of the conversation. The model indicates that there are two ways of doing this, either by directly reducing the costs incurred by the agents (e.g. subsidies) or by changing the competitive environment they are in. As the total set of non-viable points is increasing in m, reducing the costs associated with the investment process is also key in increasing the number of conversations. Doing this in settings where conversation has high relative returns or is slow (high β) will further increase the likelihood of such exchanges materializing.

The results support several hypotheses derived from the observations of collaboration. As Bikard et al. (2015) note, cross-departmental academic collaboration tends to be among the most successful. Collaborators may be willing to exert more effort and thereby incur larger costs when the degree of competition with their partner is low. Low costs associated with the investment decision may induce conversation. Pool et al. (2013) hypothesize that the effect on relative performance of sharing some stock picks may not be large and so fund managers may participate in the exchange of good ideas, a hypothesis weakly supported by the result.

As the focus has been on a symmetrical setting, I cannot comment on whether

[7] refer to Ganglmair et al. (2016) for a derivation

[8] e.g. an improved technology from joint R&D or a more elaborate theory on a scientific topic

different skill levels can be chosen in equilibrium. This may occur in a setting with asymmetrical market sizes or cost functions, allowing for different best-response curves and possible equilibria not on the symmetrical 45 degree line. Incorporating these characteristics may shed more light on how to encourage more collaborative conversation.

4.3 Limitations and Improvements

The extension has some limitations, despite the consistency and clarity of results. The most obvious is associated with the parameter β. Within the utility function $v(s)$, a high β increases the relative incremental gain from each stage in the conversation but decreases the absolute contributions. This can be interpreted to mean that conversation is slow, as more steps are needed in order to attain the same benefit as with a lower value for this parameter. This, in turn, could be an indicator of a very complicated process, with input from both sides being necessary and slow. β appears within the cost function $c(p)$ as well. Here, a high β reduces costs for any given skill level selected by the agent. While there is a consistent interpretation of these two effects, which is that a slow process (low contributions each round) is associated with a cheaper cost of investment, when interpreting a high β as a complex and costly process, this consistency may disappear.

As stated above, 'full' participation is not really tractable. What has been named 'full' participation is an equilibrium which most likely will not ever be observed; while exerting maximum effort (i.e. incurring full cost) is a realistic premise, ensuring a 100% success probability in a project is not possible. This occurrence is another problem arising from the assumed cost function which allows for the selection of a 100% success probability. A more realistic approach may include the characteristic $\lim_{p\to 1} c(p) = \infty$, which would limit the set of equilibrium points to interior solutions.

Furthermore, while the intuition behind costly skill acquisition is clear, the model assumes a simplified form of this characteristic. In this model, a process of costly investment can be thought of as an agent committing to a certain level of effort in equilibrium and incurring the costs of commitment prior to the game commencing. Yet this fails to capture the dynamic aspect of competitive games, where agents continuously adjust their investment and skill levels. A more realistic and generalized approach may allow for continuous investment in skill with periodically updated beliefs. Depreciation of skill may also be an extension with much more realism. However, these assumptions would complicate the conditions for conversational sustainability, as the investments are thought of as sunk costs during conversation in the presented model but would constitute an active decision in every period in a multi-period investment setup.

Finally, this extension fails to capture the aspect of communication costs, which may hinder or halt the conversation process. While a high β increases the likelihood of conversational sustainability, it also implies a slower conversation, as every iteration of the idea conveys a small share of information. As such, while conversation is more likely to flow, the number of ideas needed to reach the same level of benefit as with a lower β is increased. If conveying an idea to the opposing player is associated with high costs, it is unclear whether this gradual nature of conversation will remain sustainable.

5 Conclusion

Knowledge sharing among competitors is a phenomenon that has been observed for some time. Much of this sharing is hypothesized to take place in the form of conversation, that is, bilateral communication. Intuition may lead to the belief that such exchanges are irrational, but conversation can take place truthfully between rational actors and be incentive-compatible.

This thesis investigates why and when conversation may arise between competitors. Factors influencing the costs and benefits of participating in conversation, such as the skill of the agents involved or the expectation of reciprocity, play an important role in the individual decision to participate in conversation. Fostering cooperation and collaboration is vital for the success of many projects and the positive externalities resulting therefrom.

I unify existing models on bilateral conversation with costly investments by the competing agents, thereby introducing a strategic aspect to conversation. This simple extension of Stein (2008) shows results consistent with previous models. Conversation is more likely to occur sustainably if the degree of competition between parties is low or if the conversation is of a gradual nature. These factors, along with the general cost structure associated with the process, influence the degree to which conversation is sustainable. They also lead to different levels of investment by the agents in equilibrium, with lower costs or competition and a more gradual nature to conversation inducing agents to increases their optimal skill level. An increase in skill implies a longer and more productive process. These results remain consistent with the empirical literature on observed collaborative processes between competing agents.

There is, perhaps surprisingly, no incentive to free-ride in this setting, as input from agents is of a complimentary nature. It is also in the best-interest of the agents to participate in any sustainable collaborative process, as the payoffs associated with these are weakly higher than those of non-participation. This result must be used cautiously however, as the costs of communication itself are not present within this framework.

Future work in this field should allow for a more general approach compared to his thesis. When testing this hypothesis with data, the model may have to be modified to allow for an idea generation process differing from strict complimentarity. This assumption is in itself a limitation not representative of reality; instead of agents being wholly dependent on inputs from others, it is likely that these collaborative processes lead to efficiency gains. Nonetheless, this thesis serves as a consistent albeit simple explanation for observed exchanges between competitors.

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Appendices

A Derivations

A.1 Derivation of equation (2)

Equation (2) denotes the expected utility of sharing the idea in period t, conditioned on both players sharing in every period after period t. Therefore, both players will end up with the same number of signals in this expectation. This term is formed by the sum of each possible payoff multiplied the probability of that payoff being achieved.

In period t, a player, denoted without loss of generality as player 1, is in possession of idea t. By sharing this idea with the opposing agent 2, both players are in possession of it and the probability that the games ends with both players in possession of this idea is $\eta_t = 1 - E_i$. This is the belief 1 has that 2 will fail to come up with the next idea. Instead, for the game to end with both players having t + 1 signals, the 2 must successfully come up with the idea t + 1 and 1 must then fail to iterate further. Therefore the probability of the game ending at t + 1 signals is $\eta_{t+1} = E_i(1 - p_i)$. This can be continued for t + n signals and sum up the benefit of each idea v(n) with the probability of achieving this idea, so that the total expected utility is

$$\sum_{k=0}^{\infty} \eta_{t+k} v(t+k) = (1 - E_i)v(t) + E_i(1 - p_i)v(t+1) + E_i p_i (1 - E_i)v(t+2) + \ldots$$

$$= \sum_{k=0}^{\infty} (p_i E_i)^k [(1 - E_i)v(t+2k) + E_i(1 - p_i)v(t+1+2k)]$$

As both players always share their idea in this expectation and end up with the same number of signals, the competitive profit is abandoned and this expected gain is applied for the individual monopoly market portions. Incorporating this by multiplying this sum by $(1 - \theta)$ yields equation (2).

A.2 Derivation of equation (5)

An agent expects to be first-mover with probability one half and second-mover with probability one half. This can be used to calculate the expected utility of the agents in period 0

For a first-mover, the expected utility can be gained by inserting $t = 1$ into equation (2). This is because the first idea is guaranteed to be endowed and therefore, the expected utility of always sharing idea $s = 1$ can be used.

$$EU^{FM} = EU_i^{t=1}(p_i; p_j) = (1-\theta)(1-\beta\frac{1-p_j+p_j(1-p_i)\beta}{1-p_ip_j\beta^2})$$

The expected utility of the second-mover is not as straightforward. The second mover receives idea $s = 1$ from the opposing agent and has probability p_i to improve upon it. If successful, the expected utility of sharing is computed by inserting $t = 2$ into equation (2). However, in the case that he is unsuccessful, which happens with probability $1 - p_i$, the payoff from 1 idea, $(1-\theta)(1-\beta)$, is immediately realized. Simply inserting $t = 2$ into equation (2) is not sufficient, as (2) is conditioned on the idea already having been conceived. The expected utility of a second-mover is therefore given by:

$$EU^{SM} = p_iEU_i^{t=2}(p_i; p_j) + (1-p_i)(1-\theta)(1-\beta)$$

which can be rewritten as

$$(1-\theta)(1-\beta\frac{1-p_i+p_i(1-p_j)\beta}{1-p_ip_j\beta^2})$$

The agent's objective function is now the expected utility, consisting of the weighted sum of the terms corresponding to first- and second-mover, minus the costs. This is given by the equation:

$$\begin{aligned} EU_i^{t=0} &= \frac{1}{2}[EU^{FM} + EU^{SM}] - c(p_i) \\ &= \frac{1-\theta}{2}(2-\beta\frac{2+p_j(\beta-1)+p_i(\beta-2p_j\beta-1)}{1-p_ip_j\beta^2}) - c(p_i) \end{aligned}$$

A.3 Best Response Curves

The first-order-condition of equation (5) set to 0 gives the BRC of each agent. This is given by the following equation:

$$(1-\theta)\beta\frac{(1+\beta p_j)^2}{2(1-p_ip_j\beta^2)^2} = mp_i\frac{2-p_i\beta}{(1-p_i\beta)^2}$$

This is the implicit equation of $p_i(p_j)$ for all $p_j \in (0, 1]$. If, within this interval, $p_i > 1$, then instead of this equation $p_i = 1$

A.3.1 Proving $\lim_{p_j \to 0} p_i(p_j) > 0$

The rearranged limit is the quadratic function:

$$(1-\theta)\beta(1-p_i\beta)^2 = 2mp_i(2-p_i\beta)$$

which has two roots, the smaller of which is given as:

$$p_i = \frac{1-\beta\sqrt{\frac{2m}{2\beta^2 m-\beta^4(\theta-1)}}}{\beta}$$

This is larger than 0 for all specified ranges of the parameters.

A.4 Symmetrical Interior Solution

The agent's optimal decision can be found at the intersection of the two BRCs. Due to the symmetrical property of these functions, setting $p_j = p_i = p$ in agent i's BRC yields the same result (intersection with the 45 degree line from the origin). Doing this simplifies the equation to:

$$(1-\theta)\beta = 2mp(2-p\beta)$$

which is a quadratic equation in the solution p. Solving gives two possible points for p, denoted

$$p_1^{SE} = \frac{2m-\sqrt{2m(\beta^2(\theta-1)+2m)}}{2\beta m}$$
$$p_2^{SE} = \frac{2m+\sqrt{2m(\beta^2(\theta-1)+2m)}}{2\beta m}$$

Since the interior solution must have the property that $p \in (0,1)$ is possible, p_2^{SE} cannot be the equilibrium as:

$$\frac{2m+\sqrt{2m(\beta^2(\theta-1)+2m)}}{2\beta m} = \frac{1}{\beta} + \frac{\sqrt{2m(\beta^2(\theta-1)+2m)}}{2\beta m} \geq 1$$

for all $\beta \in (0,1)$. Noting this, $p^* = p_1^{SE}$ is the interior solution. For this solution to exist mathematically, the term under the root must be positive. This is the case if the condition

$$2m \geq (1-\theta)\beta$$

holds, which is true if condition (11), the condition for an interior solution, is valid. This solution has the characteristic of

$$p^* > 0 \ \forall \ \theta, \beta \in (0,1), \ \ \beta \geq \theta$$

Checking whether this result is larger than 1 yields equation (10), the condition for 'full' participation. Checking whether it fulfills condition (4) results in equation (11), the incentive compatibility constraint.

A.5 Derivatives

This subsection will provide the derivatives used for analysis, in a form such that it is easy to see their sign for all β, $\theta \in (0,1)$ and $m > 0$.

$$\begin{aligned}
\frac{\partial \underline{m}}{\partial \theta} &= -\frac{\beta}{2(2-\beta)} < 0 \\
\frac{\partial \underline{m}}{\partial \beta} &= \frac{1-\theta}{(2-\beta)^2} > 0 \\
\frac{\partial \overline{m}}{\partial \theta} &= -\beta^2 \frac{(1-\theta)^2+1}{2(2-\theta)^2\theta^2} < 0 \\
\frac{\partial \overline{m}}{\partial \beta} &= \beta \frac{1-\theta}{(2-\theta)\theta} > 0
\end{aligned}$$

The partial derivatives for p^*, the interior solution, can be simplified to the following:

$$\begin{aligned}
\frac{\partial p^*}{\partial \theta} =& \frac{-1}{2\sqrt{2m(2m-(1-\theta)\beta^2)}} < 0 \\
\frac{\partial p^*}{\partial m} =& \frac{-(1-\theta)\beta}{2m\sqrt{2m(2m-(1-\theta)\beta^2)}} < 0 \\
\frac{\partial p^*}{\partial \beta} =& \frac{\sqrt{2}m - \sqrt{m(2m-(1-\theta)\beta^2)}}{\beta^2\sqrt{m(2m-(1-\theta)\beta^2)}} > 0 \text{ if } (1-\theta)\beta^2 > 0
\end{aligned}$$

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